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EVALUATION OF LIGHTNING HORIZONTAL ELECTRIC FIELDS OVER A FINITELY CONDUCTING GROUND

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Abstract - Lightning electromagnetic fields radiated during the return stroke phase may induce overvoltages on overhead conductors. These transients can damage sensitive electronic equipment and provoke line flashovers, thus degrading the performance of distribution and telecommunication networks. This paper presents and discusses the characteristics of the horizontal component of the electric field produced by negative cloud-to-ground flashes during the return stroke phase. The analysis considers the influences of the distance between the return stroke location and the observation point, the soil type and the stroke current propagation velocity. The TL model is adopted for the determination of the current distribution along the return stroke channel, whereas the effect of the finite ground conductivity is taken into account by using the Cooray-Rubinstein approximation. The results show that, regardless of the ground conductivity, the distance from the lightning strike point has a great influence on the characteristics of the horizontal electric field, especially on its amplitude. For short distances to the lightning channel, the field amplitude tends to decrease with the ground conductivity, while for distances of a few hundred meters the absolute value of the horizontal electric field magnitude tends to increase as the ground conductivity diminishes.

1 INTRODUCTION

Lightning usually causes a significant amount of outages on transmission and distribution lines, contributing to the decrease of quality indexes. A great deal of problems on distribution systems is related to indirect strokes due to the voltages induced by the electromagnetic fields associated with the current propagation along the lightning channel. In order to understand the behaviour of these induced voltages it is important to evaluate the characteristics of the lightning fields in various situations. While the approximation of a perfectly conducting ground is justified for the calculation of both the horizontal magnetic and vertical electric field components for distances not exceeding a few tens of kilometers, it generally does not apply to the horizontal electric field, especially in the case of high resistivity soils. The

horizontal component of the electric field can be accurately obtained, at any height, through the resolution of the Sommerfeld integrals. By using this formulation, Zeddami and Degauque present in [1] lightning horizontal electric fields calculated at different situations. However, as pointed out in [2, 3], the solution of the exact Sommerfeld equations is computationally inefficient when applied to the lightning case as it requires an excessive CPU running time.

Several approximations have been proposed for the calculation of the Sommerfeld integrals [2, 4 - 6]. For the computation of lightning induced voltages on overhead lines, the Cooray-Rubinstein approach [2] is widely utilized in literature and permits to obtain satisfactory results for a large range of distances [2, 3, 7].

In this paper the static, induction and radiation components of the horizontal electric field E_r are evaluated with regard to the distance between the stroke location and the observation point. The influences of the soil parameters and of the stroke current propagation velocity on the field E_r are also discussed.

2 METHODOLOGY OF CALCULATION

2.1 Stroke Current

The current at the channel base at any time $i(0,t)$ is obtained by the sum of the two functions, and represents a typical subsequent stroke current [8]:

$$i(0,t) = \frac{I_{01}}{\eta} \cdot \frac{(t/\tau_1)^n}{(t/\tau_1)^n + 1} \cdot e^{-t/\tau_2} + I_{02}(e^{-t/\tau_3} - e^{-t/\tau_4}) \quad (1),$$

where $I_{01} = 9.9$ kA, $\eta = 0.845$, $\tau_1 = 0.072$ μ s, $\tau_2 = 5.0$ μ s, $I_{02} = 7.5$ kA, $\tau_3 = 100$ μ s and $\tau_4 = 6.0$ μ s. It is characterized by a peak value of about 11 kA and maximum rate of rise of about 105 kA/ μ s. In order to determine the spatial-temporal distribution of the current

along the channel $i(z', t)$, several return stroke models have been proposed [8 - 11]. In the present study the transmission line model (TL) [9] is adopted, so that the current at height z above ground level at instant t is given by

$$\begin{aligned} i(z, t) &= i(0, t-z/v) \quad \text{for } z \leq v.t \\ i(z, t) &= 0 \quad \text{for } z > v.t, \end{aligned} \quad (2)$$

where v is the stroke current propagation velocity.

2.3 Lightning Electromagnetic Fields

A straight vertical channel with length (H) of 4 km is assumed for the calculation of the lightning electromagnetic fields. The geometrical parameters used for determination of the lightning electromagnetic fields over a perfectly conducting ground are presented in Fig. 1, where h is the height of the observation point, $hc(t)$ is the actual height of the current wavefront at instant t , z' is the height of the current wavefront seen at the observation point P, and dz' is an infinitesimal portion of the lightning channel. The parameters r and $R(z')$ represent the distances from the observation point to the channel and to the position of the stroke current wavefront (seen from P), respectively.

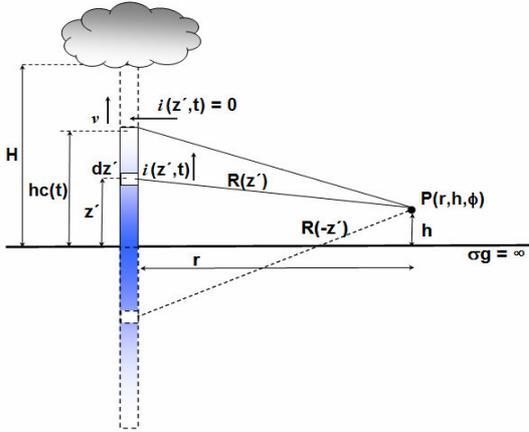


Fig. 1 - Geometrical parameters used for calculation of lightning electromagnetic fields over a perfectly conducting ground.

The horizontal electric and magnetic fields E_r and H_ϕ over a perfectly conducting ground are calculated in the time domain using cylindrical coordinates and the Master and Uman equations [12].

$$\begin{aligned} dE_r(r, \phi, h, t) &= \frac{dz'}{4\pi\epsilon_0} \left[\frac{3r(h-z')}{R(z')^5} \int_0^t i(0, \tau - z'/v - R(z')/c) d\tau \right. \\ &\quad \left. + \frac{3r(h-z')}{cR(z')^4} i(0, t - z'/v - R(z')/c) + \frac{r(h-z')}{c^2 R(z')^3} \frac{\partial i(0, t - z'/v - R(z')/c)}{\partial t} \right] \end{aligned} \quad (3)$$

$$\begin{aligned} dH_\phi(r, \phi, h, t) &= \frac{dz'}{4\pi} \left[\frac{r}{R(z')^3} i(0, t - z'/v - R(z')/c) \right. \\ &\quad \left. + \frac{r}{cR(z')^2} \frac{\partial i(0, t - z'/v - R(z')/c)}{\partial t} \right] \end{aligned} \quad (4),$$

where ϵ_0 and c are, respectively, the permittivity and the speed of light in free space. Taking into account the effect of perfectly conducting ground, the electric and magnetic fields of the image are obtained by substituting $R(z')$ for $R(-z')$ and z' for $-z'$ in equations (3) and (4). The total magnetic and electric fields are obtained by integrating the equations along the lightning channel and its image. The horizontal component of the electric field in the case of finite ground conductivity (σ_g) is calculated using the Cooray-Rubinstein approach, which can be written in terms of the electric field components as:

$$\begin{aligned} E_r(r, h, j\omega) &= E_{r, \text{elet}}(r, h, j\omega) + \\ &\quad \left(-H_{\phi, \text{ind}}(r, 0, j\omega) \cdot \frac{c\mu_0}{\sqrt{\epsilon_{rg} + \sigma_g/j\omega\epsilon_0}} + E_{r, \text{ind}}(r, h, j\omega) \right) + \\ &\quad \left(-H_{\phi, \text{rad}}(r, 0, j\omega) \cdot \frac{c\mu_0}{\sqrt{\epsilon_{rg} + \sigma_g/j\omega\epsilon_0}} + E_{r, \text{rad}}(r, h, j\omega) \right) \end{aligned} \quad (5).$$

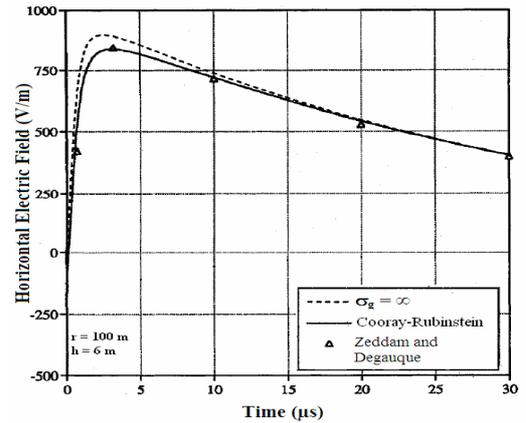
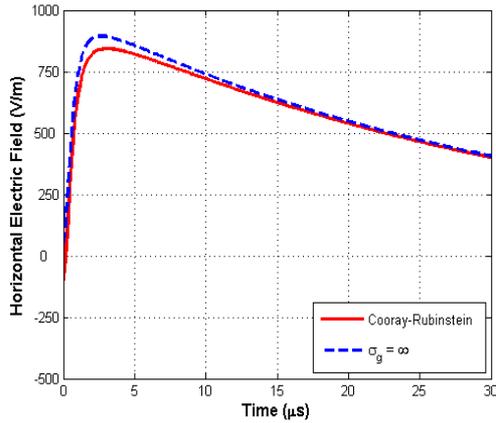
In equation (5) the first, second and third terms are referred to as the static, induction and radiation components, respectively. It is worth mentioning that the Cooray-Rubinstein approach can be derived as a special case from the formulation used by Shoory et al. [13].

In the following sections, frequency domain calculations are carried out with 1024 samples up to 8.53 MHz, which corresponds to a sampling interval of 0.0586 μs over a time window of 60 μs . The 4 km long lightning channel is divided into 4000 segments, each with the length of 1 m.

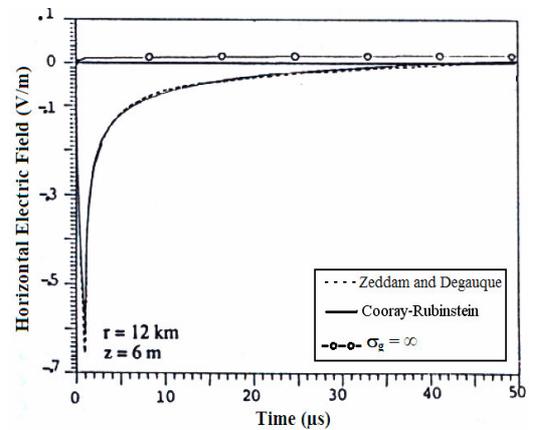
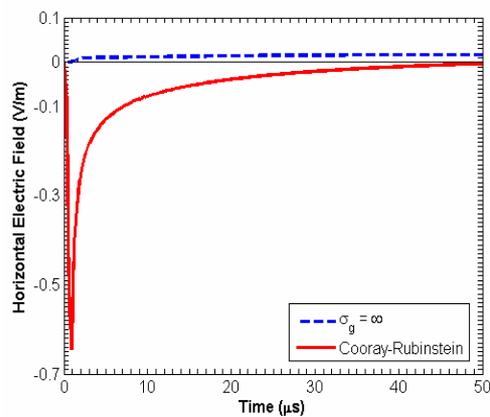
3 RESULTS AND DISCUSSION

At first, comparisons are made between the horizontal electric fields calculated according to the procedure presented in Section 2 and those presented in [3, 7]. The results relative to the distances of $r = 100$ m and $r = 12$ km are presented in Fig. 2. Then the influences of the distance between stroke location and observation point, soil type and stroke current propagation velocity, upon the horizontal electric field, are evaluated.

Unless otherwise specified, in the simulations v is assumed constant with height and equal to 90 m/ μs . whereas the height of the observation point (h) is 10 m.



(a)



(b)

Fig. 2 – Comparisons between the horizontal electric fields calculated according to the procedure described in Section 2 (left column) and those presented in [3, 7] considering two values for the ground conductivity ($\sigma_g = \infty$ and $\sigma_g = 0.01$ S/m). $\epsilon_{rg} = 10$; height of the observation point $h = 6$ m.

a) $r = 100$ m, right column: taken from [3]

b) $r = 12$ km, right column: taken from [7]

3.1 Distance to the Lightning Strike Point (r)

In order to evaluate the influence of the distance r on the characteristics of the horizontal electric field E_r , the static, induction and radiation components were calculated separately, at distances of 100 m, 1 km and 10 km from the stroke location.

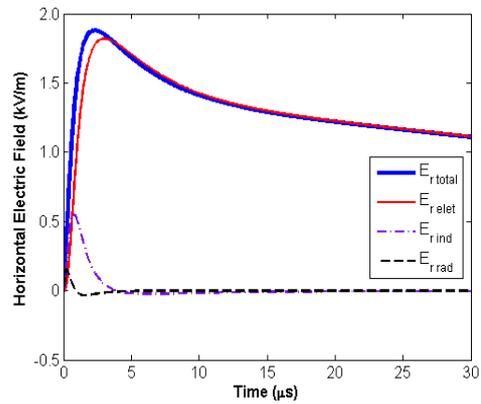
Fig. 3 presents the field E_r at 10 m above ground level, assuming the ground as a perfect conducting plane. At short distances ($r = 100$ m, Fig. 3a) the static component dominates, while the radiation component is negligible. A slight contribution to the field peak is given by the induction component. On the other hand, for large distances from the stroke channel ($r = 10$ km, Fig. 3c), the first peak is determined by the radiation component. Afterwards the contribution of the induction component increases and eventually it predominates, leading to a continuous increase of E_r in the time window considered in the simulations (30 μ s). For intermediate distances

($r = 1$ km, Fig. 3b), firstly the induction and radiation components have a greater influence, but after some microseconds both of them decrease and the static component dominates.

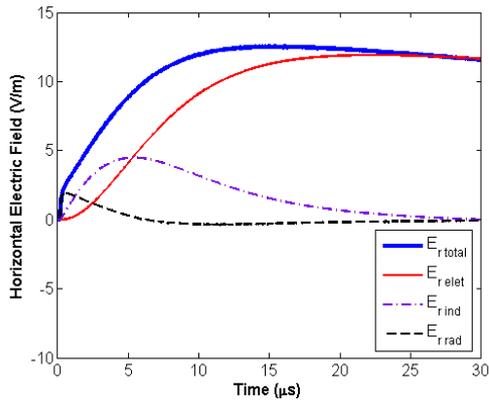
The calculations presented in Fig. 4 refer to the case of $\sigma_g = 0.02$ S/m and $\epsilon_{rg} = 30$. It can be noticed, by comparing Fig. 4a and Fig. 3a, that the influence of the finite ground conductivity is very small, illustrating that for short distances from the stroke location the assumption of the ground as a perfectly conducting plane is valid provided that the conductivity is not too low. Eq. (5) shows that, contrasting with the E_r induction and radiation components, the static one is not affected by the finite ground conductivity, regardless of the distance r .

The radiation component is the first to reach its maximum and after that its absolute value decreases abruptly. At short distances from the lightning channel it has positive polarity and its magnitude is low in comparison with the

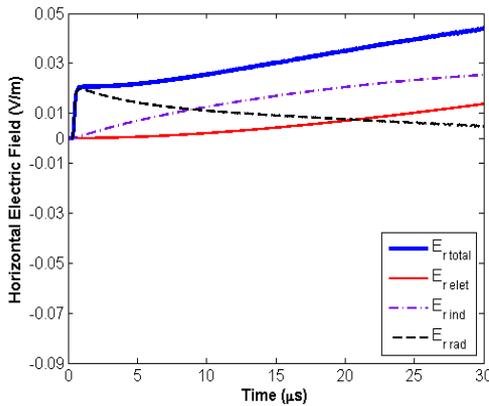
other components. However, its polarity changes to negative and its absolute value increases with the distance in such a way that for intermediate distances from the stroke location ($r = 1$ km, Fig. 4b) it dominates at the very beginning, causing a first negative peak on E_r . As it falls sharply after the peak, the static component, which is positive, soon predominates and as a consequence E_r presents a bipolar waveshape.



(a)



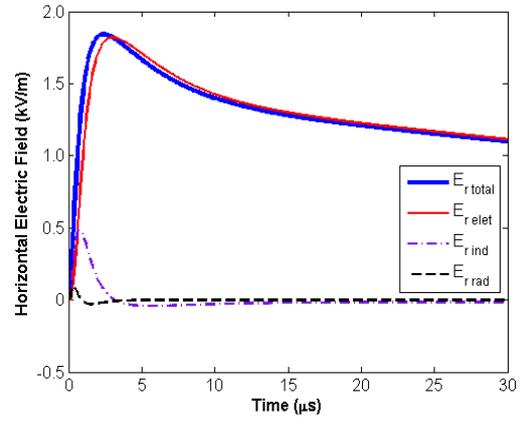
(b)



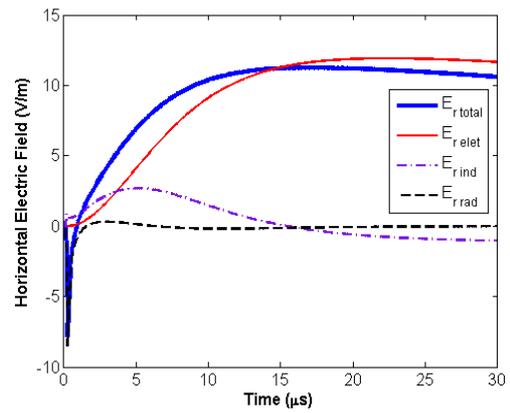
(c)

Fig. 3 – Horizontal electric field components at different distances from the lightning strike point. $h = 10$ m, $v = 90$ m/ μ s, $\sigma_g = \infty$.
a) $r = 100$ m b) $r = 1$ km c) $r = 10$ km.

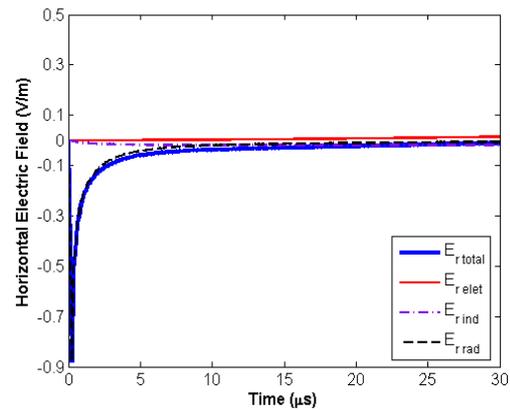
After some time (about 13 μ s in the situation illustrated in Fig. 4b) the induction component changes its polarity from positive to negative and so the total horizontal field E_r presents a slightly faster decay than the static component.



(a)



(b)



(c)

Fig. 4 – Horizontal electric field components at different distances from the lightning strike point. $h = 10$ m, $v = 90$ m/ μ s, $\sigma_g = 0.02$ S/m, $\epsilon_{rg} = 30$.
a) $r = 100$ m b) $r = 1$ km c) $r = 10$ km.

For longer distances ($r = 10$ km, Fig. 4c) the radiation component predominates all the time and for this reason the absolute value of E_r may be larger in the case of low conductivity soils. This can be clearly observed from a comparison of Fig. 3c and Fig. 4c. Not only the field magnitude but also its waveshape is significantly affected by the finite ground conductivity.

3.2 Soil Type

The ground conductivity for the same soil type varies as function of its temperature, moisture content and other parameters. In order to evaluate the influence of the soil parameters on the horizontal electric field, three soil types, taken from [14], were considered, as indicated in Table 1. For each soil type, simulations were performed considering the limits of the ground relative permittivity (ϵ_{rg}) given in Table 1. For both the good and poorly conductive ground, the variations of the field E_r were found to be negligible within the ranges indicated. On the other hand, for the case of very poorly conductive ground, the difference between the first peak (negative) corresponding to the cases of $\epsilon_{rg} = 1$ and $\epsilon_{rg} = 3$ was about 12 %. Thus, the simulations presented in this Section relative to this type of soil were done assuming $\epsilon_{rg} = 2$.

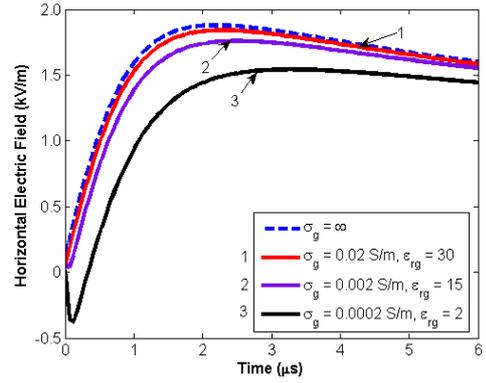
Table 1: Soil types considered in the simulations.

SOIL	σ_g (S/m)	ϵ_{rg}
Good conductive ground	0.02	4 - 30
Poorly conductive ground	0.002	10 - 15
Very poorly conductive ground	0.0002	1 - 3

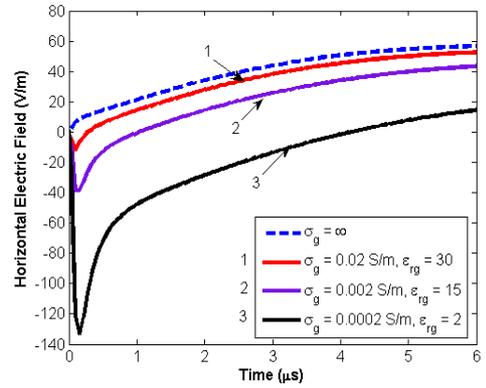
Fig. 5 shows the results corresponding to field calculations at distances of 100 m and 500 m from the lightning strike point for the three soil types considered, as well as for the case of a perfectly conducting ground.

For short distances to the stroke location (Fig. 5a), the field associated with the good conductive ground is very close to that calculated assuming the ground as a perfect conducting plane and also relatively close even to the field corresponding to the poorly conductive ground. The latter, however, presents a small negative peak in the very beginning of the waveshape. The amplitude of this negative peak increases as the ground conductivity diminishes, and the field corresponding to the case of very poorly conductive ground cannot be calculated with the assumption of $\sigma_g = \infty$.

As the distance r increases, the influence of the radiation component tends to predominate. This influence increases as the ground conductivity diminishes. Fig. 5b illustrates this situation for the case of $r = 500$ m. It can be readily seen that the lower the ground conductivity, the larger the absolute peak value of the horizontal electric field E_r .



(a)



(b)

Fig. 5 - Horizontal electric fields for three soil types at different distances from the stroke location. $h = 10$ m, $v = 90$ m/ μ s.

a) $r = 100$ m b) $r = 500$ m

3.3 Stroke Current Propagation Velocity (v)

The influence of the stroke current propagation velocity was investigated by the calculation of the horizontal electric field at the distance of 300 m from the stroke location and the height of 10 m above ground level. The ground was assumed as good conductive and two values were considered for v : 25 % and 50% of the speed of light in free space. The stroke current propagation velocity may have a remarkable influence upon E_r , as illustrated in Fig. 6. In both cases the radiation component dominates at the very beginning, and that is the reason for the initial negative peaks. The greater the propagation velocity, the larger the amplitude of the negative peak. However, the influences of the induction and static components increase with time and soon the field E_r becomes positive. The field corresponding to the higher velocity ($v = 150$ m/ μ s) is larger up to approximately 1.5μ s (neglecting the delay time); afterwards it increases little with time and reaches its crest value at about 3μ s. On the other hand, the field corresponding to the lower propagation velocity ($v = 75$ m/ μ s) continues to increase up to about 5.7μ s and eventually reaches a magnitude approximately 80% greater. In the situations considered (good conductive

ground and point of observation at 300 m from the stroke location), the larger influence upon the horizontal electric field is exerted by the static component.

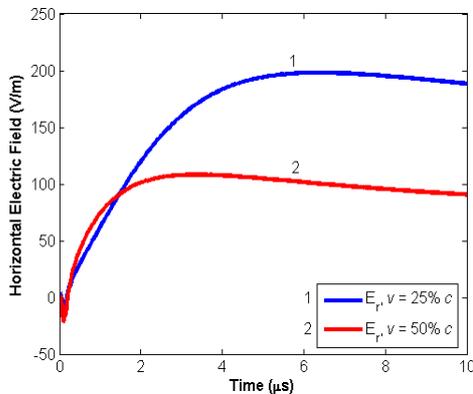


Fig. 6 - Horizontal electric field for different stroke current propagation velocities (v).

$h = 10$ m, $r = 300$ m, $\sigma_g = 0.02$ S/m, $\epsilon_{rg} = 30$.

Curve 1: $v = 75$ m/ μ s (25 % c); Curve 2: $v = 150$ m/ μ s (50% c)

4 CONCLUSIONS

The horizontal electric field E_r produced by negative cloud-to-ground flashes during the return stroke phase has been evaluated with respect to the influences of the distance between the stroke location and the observation point, soil type and stroke current propagation velocity.

The results of the simulations have shown the great effect of the soil type upon E_r . Concerning the distance from the lightning strike point, it has an important influence on the characteristics of the horizontal electric field, especially on its amplitude, regardless of the ground conductivity. For short distances to the lightning channel, the field amplitude tends to decrease with the ground conductivity, while for distances of a few hundred meters the absolute value of horizontal electric field magnitude tends to increase as the ground conductivity diminishes.

The stroke current propagation velocity may have a remarkable influence on the characteristics of the horizontal electric field.

Although the TL model used in this investigation is recommended due to its simplicity for estimates of the initial field peak from the current peak or conversely the current peak from the field peak, it is not physically plausible as it does not consider any charge removal from the lightning channel. It is possible that this limitation may somewhat influence the results; further investigations are currently being carried out on the influence of the return stroke model on the horizontal electric field.

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